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Theoretical Analysis of Polarized Structure Functions

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Abstract

We review the analysis of polarized structure function data using perturbative QCD at NLO. We use the most recent experimental data to obtain updated results for polarized parton distributions, first moments and the strong coupling. We also discuss several theoretical issues involved in this analysis and in the interpretation of its results. Finally, we compare our results with other similar analyses in the recent literature.

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1 Introduction

The interest in polarized deep inelastic scattering was revived in 1988 by the results of the famous EMC experiment [1] that led to the so-called "spin crisis" problem. Since then a lot of progress has been made. Several experiments were completed at CERN [2, 3] and SLAC [4]-[8] on proton, deuterium and ^{3}He targets. The HERMES experiment is under way at DESY [9]. Other experiments are planned with the goal of improving our knowledge of spin structure functions: COMPASS at CERN, HERA with polarized beams at DESY and RHIC at Brookhaven. On the theory side, the "spin crisis" was immediately recognised not to be a fundamental problem but rather an interesting property of spin structure functions to be understood in terms of QCD. A number of dynamical mechanisms have been proposed and studied [10, 11, 12, 13, 14, 15]. It is only at the naive parton level that the first moment of the singlet part of the structure function g_1 corresponds to the total helicity fraction carried in the target by parton quarks. However in perturbative QCD this identification is no longer valid, even approximately, due to the effect of the axial anomaly. Extensive calculations of hard cross sections for polarized processes have been performed, and in particular the complete two loop evolution kernels are now available [16]. The Wilson coefficients for the singlet and nonsinglet first moments of g_1 are known up to three loops [17]. By now the perturbative Q^2 dependent effects are computable at the same level of accuracy for polarized and unpolarized structure functions. In addition, the study of the expected behaviour of polarized structure functions at small x has much progressed, in parallel with similar results for unpolarized parton densities motivated by the HERA experimental data.

At present, we can say that one phase of the study of polarized structure functions has been concluded. In this first phase attention was mainly concentrated on first moments and sum rules. The main conclusions are that the Bjorken sum rule is valid within one standard deviation while the Ellis-Jaffe sum rule is violated at a level of about three standard deviations. Attention is now shifting to the reconstruction of polarized parton densities at all x and Q^2. In particular one is interested in the gluon density, since this is expected to have special properties in polarized deep inelastic scattering. The possibility of inferring the gluon density from scaling violations is under active study. Of special interest is the behaviour at small values of x of all polarized parton densities and their variation with Q^2. The data gathered at HERA on the small x behaviour of unpolarized structure functions and related theoretical work together imply that a simple Regge extrapolation at small x is unreliable at large Q^2. This Regge extrapolation was used in the past to derive first moments of polarized structure functions and the values obtained are indeed significantly biased by this assumption. The lesson from the HERA experimental results is that Regge behaviour in general only applies at small $Q^2 \lesssim \Lambda^2$. At large Q^2 the behaviour induced by the QCD evolution prevails if it is more singular than the Regge prediction. Actually, for $x \to 0$, the Regge behaviour is less singular than the QCD evolution for all parton densities except the nonsinglet unpolarized quark densities. The prejudice that Regge behaviour should be valid at small x for all Q^2 values is often supported by the observation that it has some empirical success in the case of nonsinglet unpolarized quark densities. But, as we said, this case is the exception rather than the rule.

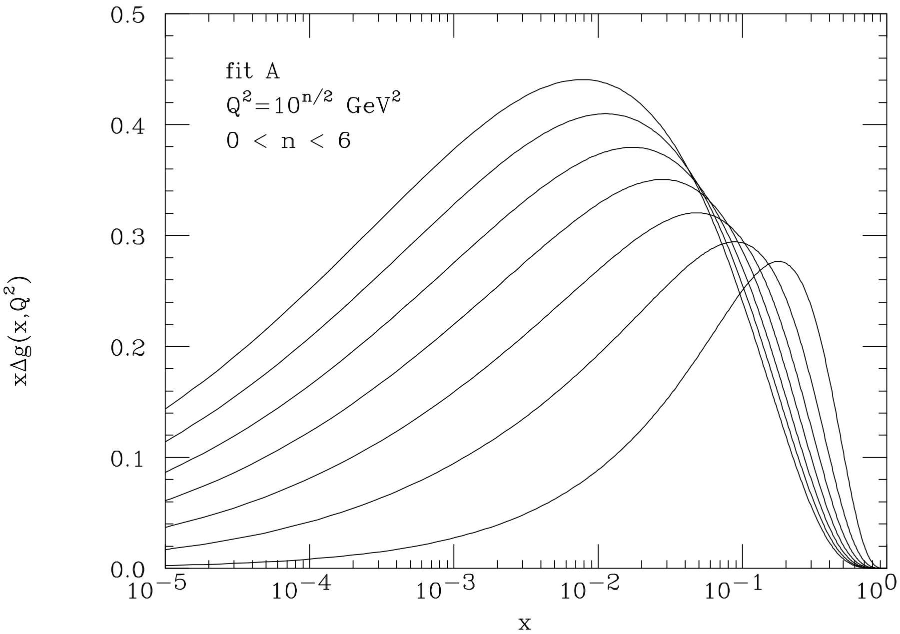


Figure 1: *Plot of* $x\Delta g(x, Q^2)$ *for fit A (see sect. 3). The curves correspond to* $Q^2 = 10^{n/2}$ *GeV*^{2}*,* $n = 0, \ldots, 6$

The gluon density is of special interest in polarized deep inelastic scattering because its first moment is predicted to increase like $1/\alpha_s(Q^2)$ while higher moments are decreasing functions of $\log Q^2$ (falling at a faster rate than for the unpolarized gluon density). The resulting behaviour in x and Q^2 is shown in fig. 1. The distribution, with growing area, is rapidly shifted to smaller x as Q^2 increases. The Q^2 behaviour of the first moment of the polarized gluon density was originally derived when the QCD evolution equations were first written down in x space. In fact the first moment of the polarized gluon splitting function is finite and proportional to the first coefficient of the beta function, which establishes the quoted relation with the running coupling $\alpha_s(Q^2)$. This relation between the Q^2 evolution of the first moment of the polarized gluon density and the running coupling is induced by the axial anomaly and corresponds to the non–renormalisation of the anomalous vertex $\alpha_s F_{\mu\nu}\tilde{F}^{\mu\nu}$.

It is well known that there is no operator that corresponds to the first moment of the polarized gluon density in the operator light cone expansion. But the gluon density and its first moment can be precisely defined in the more general context of the QCD improved parton model which is the established approach for hard processes. For example, we can use as a defining measurement the (sub)process $g\, +\, proton \rightarrow Higgs\, +\, X$ [18]. In leading order, only the gluon density in the proton contributes to the cross section:

$$\sigma(S, M_H^2) = \int dx\ g(x, M_H^2)\tilde{\sigma}(xS) = \frac{C}{S}\, g(\frac{M_H^2}{S}, M_H^2) + \ldots \tag{1}$$

Here we used the fact that in leading order the partonic cross section $\tilde{\sigma}$ for $g + g \rightarrow H$ is proportional to a delta function: $\tilde{\sigma} = C\delta(xS - M_H^2)$, with C a constant. Thus by adjusting M_H^2/S all values of x can be reached and the first moment can also be computed. Moreover, since the Higgs is a scalar, for a polarized gluon on a polarized proton only gluons in the proton with the same helicity as the external gluon can contribute in any reference frame where the proton and the incident gluon have collinear spatial momenta. We can thus separately define g_+ and g_- in terms of cross sections for physical processes, which are necessarily positive definite. The positivity condition $|\Delta g| \leq g$ is thus automatically guaranteed by the positivity of the cross section. However this identification is only valid at LO and, in a generic factorization scheme, it will be violated at NLO and beyond. As a consequence, when performing NLO fits, it is wrong to impose the positivity condition within a generic gluon definition. This unnecessarily restrictive assumption can lead to very misleading conclusions, particularly if the starting scale is chosen small, as is often the case.

We have seen that the polarized gluon density, including its first moment, can be defined from hard processes outside totally inclusive deep inelastic scattering. At leading order this definition is totally unambiguous (provided that the defining hard process is within the set of observables that obey the factorisation theorem). The first moment of the polarized gluon density, $\Delta g(1, Q^2)$, obeys the polarized evolution equations and increases like $1/\alpha_s(Q^2)$. As a consequence the definition of the singlet quark first moment becomes totally ambiguous, because two generic definitions differ by terms of order $\alpha_s(Q^2)\Delta g$. For the first moment what is formally a next to leading order correction is potentially of the same size as the leading term. As a consequence the singlet quark first moment, defined directly from the structure function g_1 (i.e. the one used by the experiments when the "spin crisis" [19] was announced) does not have to coincide with the constituent quark value (the total fraction of the proton spin carried by quarks). Only for exactly conserved quantities do the corresponding values for constituent and parton quarks have to coincide. The first moments of the quark densities are in general only conserved at leading order by the QCD evolution. But, due to the axial anomaly, the singlet quark first moment defined from g_1 is not conserved in higher orders. However it can be shown that it is possible to select a definition of the singlet quark first moment in such a way that it is conserved at all orders. The relation between the singlet quark first moments in g_1, which we will call $a_0(Q^2)$, and the conserved definition $\Delta\Sigma(1)$ is given by

$$a_0(Q^2) \;=\; \Delta\Sigma(1) \;-\; n_f \frac{\alpha_s(Q^2)}{2\pi} \Delta g(1, Q^2). \tag{2}$$

One main challenge for experiments on polarized deep inelastic scattering is to measure the polarized gluon density and its first moment with sufficient accuracy to be able to verify the above relation, that is to check that $\Delta\Sigma(1)$ is indeed compatible with its constituent value. Note that this constituent value appears to be significantly less than unity, i.e. $\Delta\Sigma(1) \sim 0.6$ [20].

The purpose of this paper is to update our recent analysis [21] of all the available data on polarized structure functions in order to extract the polarized parton densities and their moments for comparison with theoretical expectations. The differences with respect to the data sets used in ref. [21] are the following: we include the most recent SMC data with proton target [2], which differ considerably from their older analyses in the small x region; we use here the published SMC deuteron data [3] and E154 neutron data [8], which are slightly different from the preliminary data sets used in ref. [21]; and finally we include the recent data for g_1 with a ^{3}He target obtained by the HERMES Collaboration [9]. We have not included in our fits the old data of refs.[1, 22], which have much larger uncertainties than those of more recent experiments. We have checked, however, that the inclusion of these old data does not affect our results. Similar NLO analyses have been recently performed by other groups. We shall comment in the following on these papers, pointing out the differences with respect to our work.

We will discuss the possibility of obtaining the polarized gluon density (as well as the other parton densities and $\alpha_s(Q^2)$) from the observed scaling violations in the g_1 data. The result of our analysis is that there is some indication in the data for a large positive gluon component, large enough to make $\Delta\Sigma(1)$ close to the constituent value. But the uncertainties are still very large. In fact our conclusion that the data indicate a large gluon component is not shared by other authors, as we will discuss in the following. Let us mention in passing that often the following (false) argument is made to imply that it is impossible to derive Δg or $\alpha_s(Q^2)$ from the present data on scaling violations. The quantity which is directly measured is the cross section asymmetry A_1 that experimentally does not show any appreciable Q^2 dependence within the present accuracy of the data. So – the argument goes – how can it be possible to derive values of Δg or $\alpha_s(Q^2)$ different from zero from data that do not show any scaling violations? The reason why this argument is false is that asymptotically $A_1 \sim g_1/F_1$, where F_1 is the unpolarized structure function. Now the QCD evolution equations do not apply directly to A_1; rather, as is well known, two different evolution equations, with different kernels, are valid for g_1 and F_1. The approximate cancellation of the scaling violations for A_1 in the measured range is a strong constraint on the g_1 scaling violations, given that the scaling violations for F_1 are at present known with much larger accuracy. Thus it is a remarkable consistency check that the observed approximate equality of the scaling violations for g_1 and F_1, when analysed in terms of the evolution equations for g_1, do indeed lead to a value of $\alpha_s(Q^2)$ which is in agreement with the world average (and many standard deviations away from zero).

We will also show that when all the data are included it is possible to make a reliable test of the Bjorken sum rule [23]. For an experimental verification of the Bjorken sum rule one has to extract from the data the first moment of the difference of polarized up and down quark densities at some convenient value of Q^2. Data taken at all kinematically accessible values of x and Q^2, and on all available targets, contain information relevant for the reconstruction of polarized parton densities at a given Q^2 and ought therefore to be included. The complete NLO evolution kernels [16] can be used to reduce to the same Q^2 data measured at different Q^2 for each x. Since the evolution equations [24] for partons at a given x and Q^2 depend only on the values of the parton densities at larger values of x and the same Q^2, the necessary correction can only be performed through a general fit to all the data, which yields a set of polarized parton densities obeying the correct evolution equations [25, 26]. However in order to perform a fit one must start with a particular ansatz for the parton densities at some reference Q_0^2. Clearly the results of the fit will depend to some extent on the starting ansatz one adopts, and this dependence will induce an error in the computed first moments, and in particular in the Bjorken sum. Here we will devote special attention to this issue.

Once the data are reduced to a common Q^2 for all x values, an extrapolation to unmeasured values at small and large x is needed in order to obtain the first moment. The extrapolation at small x is especially important [27].[1] In most of the existing analyses, including those in most of the experimental papers, it has been performed by assuming a simple power behaviour based on Regge theory [28]. This leads to a rather small contribution to first moments from the small x region, since the expected extrapolation is at most flat.

As already mentioned, a naive Regge extrapolation is not justified if one wants to consider first moments in the perturbative region: small x contributions to first moments can be relatively large, especially as Q^2 increases. Here we will discuss alternative extrapolation procedures and the errors associated with them. Our guiding principles will be the validity of Regge predictions at low Q^2 and the buildup with Q^2 of the effect of the QCD evolution. From these starting points we will estimate the uncertainty in the small x extrapolation, which when combined with the evolution corrections and the more standard sources of error will allow us to quantify the extent to which the Bjorken sum rule may be tested using existing data. In practice we will do this by deriving from the data the value of g_A and the associated error for an appropriate range of values of α_s.

We will then consider the determination of α_s from the polarized deep inelastic scattering data. Previous attempts in this direction [29] have assumed the validity of the Bjorken sum rule, and used a value for the Bjorken integral obtained from the first moments given by the various experimental collaborations, and thus based on naive Regge extrapolation at small x. However, when the effects of perturbative evolution on the small x extrapolation are properly taken into account, the evaluations of the first moments must be revised: their errors then turn out to be considerably increased, and the determination of α_s from the Bjorken sum rule no longer works so well. However, we are able to show that a much better determination of α_s may be obtained if all the available data and not only the Bjorken integral are used in the analysis: the comparison of the data at small and large Q^2 in the measured range of x then leads to a reasonably precise measurement of α_s.

2 Polarized Structure Functions and Partons

We begin by summarising various results on the relation between structure functions and polarized parton distributions, and their behaviour at small x, which will be important for the following discussion.

[1] Note that the behaviour at small x of the input ansatz for the parton densities at Q_0^2 is not relevant for the evolution correction, which only depends on x values larger than the smallest measured one. On the contrary the integration at small x that completes a given moment is very much dependent on the small x behaviour of the input distributions, as we shall see.

2.1 Defining Polarized Parton Densities

The structure function g_1 is related to the polarized quark and gluon distributions by [26]

$$g_1(x,Q^2) = \tfrac{\langle e^2\rangle}{2}[C_{NS}\otimes\Delta q_{NS} + C_S\otimes\Delta\Sigma + 2n_f C_g\otimes\Delta g], \tag{3}$$

where $\langle e^2\rangle = n_f^{-1}\sum_{i=1}^{n} e_i^2$, $\otimes$ denotes convolution with respect to x, and the nonsinglet and singlet quark distributions are defined as

$$\Delta q_{NS} \equiv \sum_{i=1}^{n_f}(\tfrac{e_i^2}{\langle e^2\rangle}-1)(\Delta q_i+\Delta\bar{q}_i), \qquad \Delta\Sigma \equiv \sum_{i=1}^{n_f}(\Delta q_i+\Delta\bar{q}_i), \tag{4}$$

where Δq_i and $\Delta\bar{q}_i$ are the quark and antiquark distributions of flavor i and Δg is the polarized gluon distribution. The evolution equations for the polarized parton densities are given by

$$\begin{aligned} \frac{d}{dt}\Delta q_{NS} &= \frac{\alpha_s(t)}{2\pi}P_{qq}^{NS}\otimes\Delta q_{NS}, \\ \frac{d}{dt}\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix} &= \frac{\alpha_s(t)}{2\pi}\begin{pmatrix}P_{qq}^S & 2n_f P_{qg}^S\\ P_{gq}^S & P_{gg}^S\end{pmatrix}\otimes\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}, \end{aligned} \tag{5}$$

where $t=\log Q^2/\Lambda^2$. The coefficient functions C and the polarized splitting functions P are now known at LO [24] and NLO [16]. Moments of coefficient functions and parton densities are defined as $f(N)=\int_0^1 dx x^{N-1}f(x)$ and denoted by $C(N,\alpha_s)$, $\Delta q_{NS}(N,Q^2)$, $\Delta\Sigma(N,Q^2)$ and $\Delta g(N,Q^2)$.

As is well known [30], the definition of the singlet quark density $\Delta\Sigma(x,Q^2)$ must be carefully specified. In fact, the scheme dependence of its first moment is proportional to $\alpha_s(t)\Delta g(1,Q^2)$. This implies that the ambiguity in $\Delta\Sigma(1,Q^2)$ does not vanish asymptotically, because, due to the axial anomaly, $\alpha_s(t)\Delta g(1,Q^2)$ is scale independent at LO. For a sensible comparison with the constituent quark spin one must thus define $\Delta\Sigma(1,Q^2)$ in such a way that it is scale independent [10, 11]: $\Delta\Sigma(1,Q^2)$=$\Delta\Sigma(1)$. This is the definition we will adopt here. We then have

$$\Gamma_1(Q^2) \equiv \int_0^1 dx g_1(x,Q^2) = \tfrac{\langle e^2\rangle}{2}[C_{NS}(1,\alpha_s(t))\Delta q_{NS}(1) + C_S(1,\alpha_s(t))a_0(Q^2)], \tag{6}$$

with a_0 the singlet axial charge:

$$a_0(Q^2) = \Delta\Sigma(1) - n_f\tfrac{\alpha_s(t)}{2\pi}\Delta g(1,Q^2) \tag{7}$$

The higher moments of the singlet quark distribution are also scheme dependent, although in a less dramatic way. Various schemes were discussed in ref. [26, 31] and the dependence of the results of the analysis on the choice of scheme was studied. Here we do not come back to this issue but instead simply adopt the AB scheme as defined in ref. [26].

In view of the need to extrapolate the data to $x = 0$ in order to compute moments, it is important to summarise the current understanding of the small x behaviour of structure functions. For the unpolarized singlet quark and gluon distributions the QCD evolution equations (5) lead to the following asymptotic behaviour at small x [32, 33]:

$$\begin{aligned} xg &\sim \sigma^{-1/2} e^{2\gamma\sigma - \delta\zeta}\left(1 + \textstyle\sum_{i=1}^{n} \epsilon^i \rho^{i+1} \alpha_s^i\right), \\ x\Sigma &\sim \rho^{-1} \sigma^{-1/2} e^{2\gamma\sigma - \delta\zeta}\left(1 + \textstyle\sum_{i=1}^{n} \epsilon_f^i \rho^{i+1} \alpha_s^i\right), \end{aligned} \tag{8}$$

where $\xi = \log x_0/x$, $\zeta = \log\left(\alpha_s(Q_0^2)/\alpha_s(Q^2)\right)$, $\sigma = \sqrt{\xi\zeta}$, $\rho = \sqrt{\xi/\zeta}$, and the ϵ terms indicate corrections from the n-th perturbative order (with $n = 1$ corresponding to NLO). It follows that the structure functions xF_1 and F_2 rise at small x more and more steeply as Q^2 increases, though, for all finite n, never as steeply as a power of x. For all other parton distributions f ($f = q_{NS}$, $\Delta q_{NS}, \Delta\Sigma, \Delta g$) one has similarly [34, 25]

$$f \sim \sigma^{-1/2} e^{2\gamma_f\sigma - \delta_f\zeta}\left(1 + \textstyle\sum_{i=1}^{n} \epsilon_f^i \rho^{2i+1} \alpha_s^i\right). \tag{9}$$

Thus these distributions are less singular by a factor of x than the singlet unpolarized distributions eq. (8), while the higher order corrections are more important at small x since the exponent $i + 1$ is replaced by $2i + 1$; this is because the leading small N contributions to the anomalous dimensions at order α_s^{i+1} are $\left(\alpha_s/(N-1)\right)^i$ in the unpolarized singlet case, but $N\left(\alpha_s/N^2\right)^i$ for the nonsinglet and polarized distributions.

The limiting behaviour (8,9) implied by the evolution equations (5) at any finite order in perturbation theory would change if the series of higher order powers of $\log 1/x$ were summed to all orders to give a powerlike behaviour in x, which would then overwhelm the leading terms. As is well known [35], in the unpolarized singlet channel one may obtain a result as singular as $x^{-\lambda}$, with $\lambda \sim 1/2$, for $x\Sigma$ and xg by summation of higher order singularities in the Regge limit of $x \to 0$ at fixed Q^2 (hence fixed α_s). However the meaning and the value of a fixed α_s are quite ambiguous, and it is not at all necessary a priori that such a singular behaviour is of relevance in the measured HERA region. In fact the experimental results from HERA show no evidence at all for this behaviour [33, 36]. In principle the higher order terms could be more important for the nonsinglet and polarized distributions due to the $2i + 1$ exponent in eq. (9) instead of $i + 1$ in eq. (8). Indeed summing these 'double' logarithmic singularities [37, 38] appears to lead to a singular behaviour $f \sim x^{-\lambda}$ with $\lambda \sim 0.5$ for q_{NS} and Δq_{NS}, and $\lambda \gtrsim 1$ for the singlet densities (which would imply that the first moment of the singlet part of g_1 is actually divergent). If one were to take these theoretical predictions seriously, the errors in the small x extrapolations considered below, particularly in the singlet channel, would have to be considered only as lower bounds. However the summation of 'double' logarithms is even less well founded theoretically than the summation of 'single' logarithms in the unpolarized singlet channel, and we believe that at present none of these results should be taken too literally [36].

Another important difference between the small x behaviour of unpolarized and polarized singlet distributions is that in the unpolarized case only the gluon anomalous dimension carries the leading singularity, and consequently the rise in the singlet quark distribution is driven directly by that of the gluon, while in the polarized case all the entries in the matrix of singlet anomalous dimensions are singular, and the polarized singlet quark and gluon distributions mix. It turns out that the leading eigenvector of small x evolution is then such that the singlet quark and gluon distributions have opposite sign, which means in practice that the singlet component of g_1 is driven negative at small x and large Q^2 [25]. Contributions to first moments of g_1 from the small-x tail thus tend to become negative when Q^2 is sufficiently large.

The purely perturbative asymptotic predictions eqs. (8,9) only hold when the input distribution at the starting scale Q_0^2 is relatively nonsingular: if the singularity in the input is stronger than that generated perturbatively then the input will be essentially preserved by the perturbative evolution. The rise at small x will then be largely independent of Q^2, rather than becoming steeper as Q^2 increases. If we take the starting scale in the crossover region between perturbative and nonperturbative dynamics, we can presumably take the small x behaviour of the input from Regge theory. For unpolarized distributions the input to the singlet distributions (given by the pomeron trajectory) is then relatively flat, and indeed the dominance of the perturbative behaviour (8) is confirmed by F_2 data from HERA [33, 36], while the input to the nonsinglet (given by the $\rho - \omega$ Reggeon trajectory) is singular, behaving as $x^{-1/2}$, so it is preserved by the evolution and is consistent with data from NMC [39] and CCFR [40]. For polarized distributions Regge theory suggests that the form of the input should be given by the A_1 trajectory, and thus flat or even vanishing, behaving as x^0–$x^{0.5}$ [28].

In the following we will consider various scenarios which should cover the spectrum of reasonable small-x behaviors: either we will assume a physical picture, inspired by the HERA results, in which we assume the validity of Regge behaviour at small x in the soft region (i.e. that at some input scale $Q_0^2 \lesssim 1$ GeV^2 the polarized densities are flat or vanishing), while at larger Q^2 the effect of NLO perturbative evolution is superimposed (giving a perturbative growth of the form eq. (9)), or, at the opposite extreme, we will allow steeper inputs in the nonsinglet sector, provided they are not more singular than $|\Delta q_{NS}(x, Q^2)| \lesssim x^{-0.5}$ as $x \to 0$. This picture turns out to be consistent with the data, and gives a constraint on the allowed growth of $|\Delta q_{NS}(x, Q^2)|$ in the unmeasured region which in turn limits the possible ambiguity on the Bjorken sum rule from the small x extrapolation.

3 Polarized Parton Densities from g_1 Data

We now consider in detail the problem of extracting relevant physical quantities from the existing data. We devote particular attention to the study of the dependence of the results on the assumed functional form of the input parton distributions. For this purpose, we consider a variety of possible parameterizations of the input, we evolve these up to the values of x and Q^2 where data are available by solution of the evolution eqs. (5) at NLO, and we determine the free parameters of the input by a best fit of $g_1(x, Q^2)$ eq. (3) to all the data of refs. [2]-[9] with $Q^2 \geq 1$ GeV^2. Experimental data for g_1 are obtained from the experimentally measured asymmetries A_1 using a parameterization of the measured unpolarized structure functions F_2

[41] and R [42], consistently neglecting all higher twist corrections, and, for deuterium and helium targets, accounting for effects due to the nuclear wavefunction (but not Fermi motion or shadowing) by a simple multiplicative correction [43, 7]. Throughout this section we take $\alpha_s(m_Z) = 0.118 \pm 0.005$ [44] and $a_8 = 0.579 \pm 0.025$ [45] where a_8 is the SU(3) octet axial charge (in the proton, below charm threshold, $\Delta q_{NS}(1) \equiv \eta_{NS} = \frac{3}{4} g_A + \frac{1}{4} a_8$).

To begin with we parameterize the initial parton distributions at $Q_0^2 = 1$ GeV^2 according to the conventional form

$$\Delta f(x, Q_0^2) = \mathcal{N}_f \eta_f x^{\alpha_f} (1-x)^{\beta_f} (1 + \gamma_f x^{\delta_f}) \tag{10}$$

where Δf denotes Δq_{NS}, $\Delta\Sigma$ or Δg and $\mathcal{N}_f$ is a normalisation factor chosen such that the first moment of Δf is equal to η_f. The signs of all parameters are left free, including the overall factors η_f (although the data always choose η_f to be positive). It is particularly important to see to what extent the data fix the small x behaviour i.e. the exponents α_f in eq. (10). The g_1 data on neutrons show a strong fall at small x, while the deuteron data are much flatter: thus the fitted nonsinglet quark densities at small x tend rise while the singlet quarks tend to remain fairly flat. Starting from a generic input set of densities of the class eq. (10) we can thus easily end up with Δq_{NS} considerably more singular than $\sim x^{-0.5}$. However, the smallest value of x covered by data is still rather large, so whether this is actually the case will depend on how the functional form chosen for the fit extrapolates to very small x the behavior observed in the last few small x data points.

Indeed, a careful analysis reveals that there is a strong correlation between α_f and δ_f: one can easily push α_{NS} closer to zero by decreasing δ_{NS} from unity without appreciable changes in the quality of the fit. Thus we find that the existing data do not much constrain the behaviour of the nonsinglet at asymptotically small values of x: even within the simple functional form eq. (10) one still has a considerable flexibility in the asymptotic behaviour as $x \to 0$, and α_{NS} can be made to vary from values close to zero down to values of order ~ -1 by tuning the parameter δ_{NS}. The resulting uncertainty coming from the small-x extrapolation is therefore arbitrarily large, if no assumption is made on the small-x behavior of the nonsinglet distribution. A reasonable bound to the most singular admissible behavior, i.e. a lower bound on α_{NS} is provided by the behavior of the unpolarized nonsinglet distributions, namely $\sim x^{-0.5}$ for $x \to 0$, which is the hardest Regge behavior in the nonsinglet channel and it is also the behavior generated by the summation of double logs discussed in sect. 2.2. Since we will exhibit below small-x behaviors which are softer than any power, we consider a fit of this class, denoted as fit A, corresponding to such most singular behavior, namely:

$$\delta_\Sigma = \delta_g = 1, \qquad \delta_{NS} = 0.75 \ \text{(fixed)}, \qquad \beta_g = 4 \ \text{(fixed)}, \qquad \gamma_\Sigma = \gamma_g \qquad \text{(fit A)}. \tag{11}$$

The result obtained from this fit are listed in table 1.

Next, in order to discuss less singular inputs, we completely change the functional form of the input densities (while keeping the initial scale at $Q_0^2 = 1$ GeV^2). We thus choose an input in which the rise at small x is at most logarithmic (fit B):

$$\Delta\Sigma \;=\; \mathcal{N}_\Sigma \eta_\Sigma x^{\alpha_\Sigma} (\log 1/x)^{\beta_\Sigma}$$

$$
\begin{aligned}
\Delta q_{NS} &= \mathcal{N}_{NS}\eta_{NS}\left[(\log 1/x)^{\alpha_{NS}} + \gamma_{NS}x\,(\log 1/x)^{\beta_{NS}}\right] \qquad \text{(fit B)}, \\
\Delta g &= \mathcal{N}_g\eta_g\left[(\log 1/x)^{\alpha_g} + \gamma_g x\,(\log 1/x)^{\beta_g}\right]
\end{aligned}
\tag{12}
$$

The small x behavior of this fit is weaker than any power, and thus in particular compatible with the Regge prediction. Note also that $\log 1/x \sim (1-x)$ near $x=1$, so the behaviour as $x \to 1$ is taken care of by the γ terms. The results we obtain from this fit are again reported in table 1. The quality of fit B much better than fit A, as seen from the χ^2 value.

Although the logs are reminiscent of QCD evolution the functional form of fit B might perhaps appear a little ad hoc. It is thus interesting to try to generate the logarithms in a more physical way, by perturbative evolution. In this spirit we consider another set of trials, where we start the QCD evolution at a very small scale, $Q_0^2 = 0.3$ GeV^2, and fit a function of the form eq. (10). The choice of such a low scale is simply used as a trick to generate an effective set of distributions at the value of Q^2 at which we begin to fit the data, i.e. $Q^2 = 1$ GeV^2 (data with lower Q^2 being still discarded), with the logs piled up in a way entirely consistent with perturbative evolution. In table 1 we report the results from a fit with

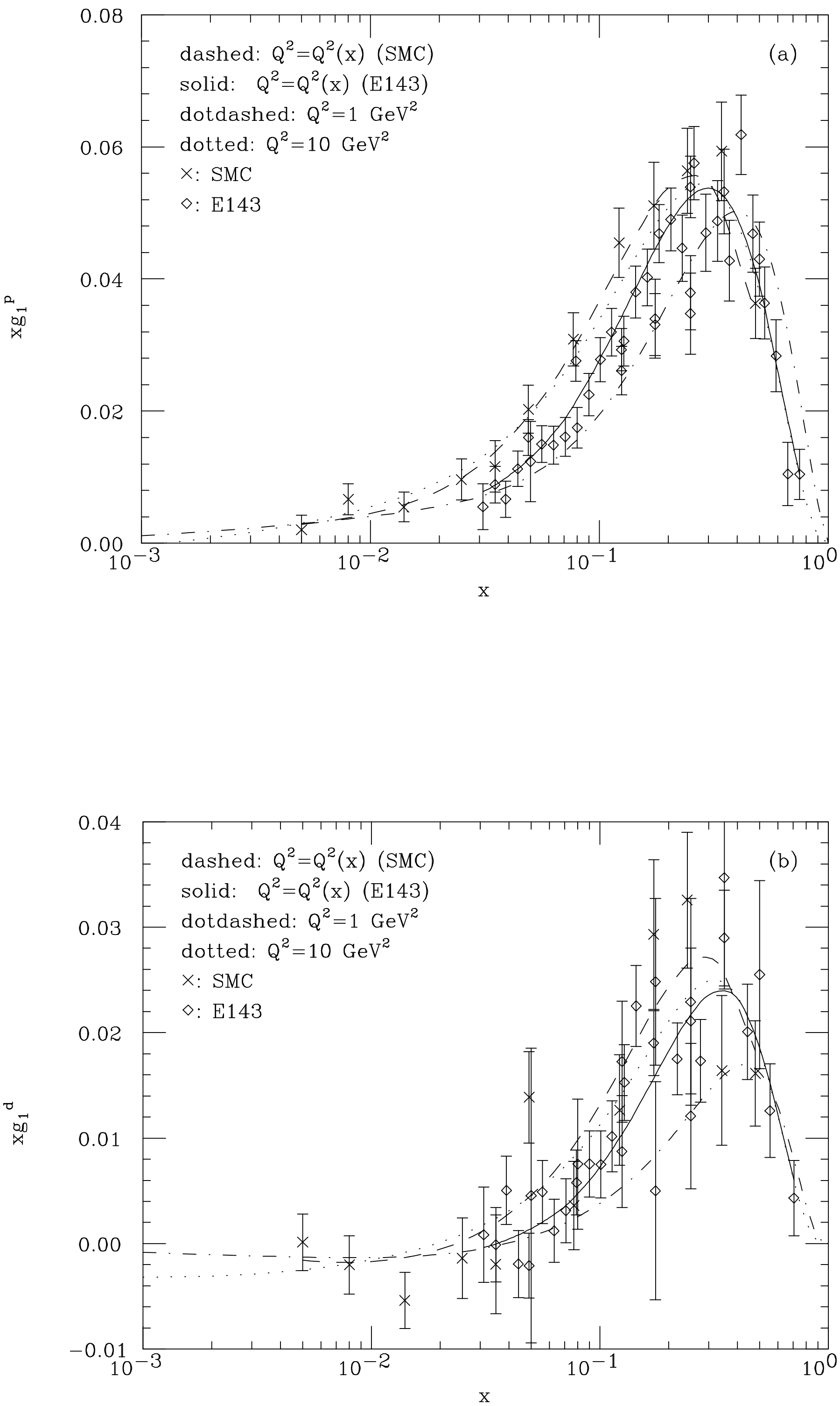
$$
\gamma_\Sigma = \gamma_g = \gamma_{NS} = 0, \qquad \beta_g = 15 \ \text{(fixed)} \qquad \text{(fit C)}. \tag{13}
$$

In this class of fits, the large-x behaviour of the gluon distribution can hardly be determined by the fitting procedure; therefore, we fixed $\beta_g = 15$ at $Q_0^2 = 0.3$ GeV^2, because we checked that this choice approximately corresponds to a $(1-x)^4$ behaviour of Δg at large x and Q^2 around 1 GeV^2. The quality of the fit in the measured region is comparable to that of fit A. Comparing with the results of fit A, we see that by lowering the initial Q_0^2 scale all the exponents in the x^α terms have become positive in qualitative agreement with the idea that naive Regge behaviour is restored at a sufficiently low scale. Indeed a fit of reasonable quality is also obtained if we fix all exponents α_f at $Q_0^2 = 0.3$ GeV^2 to the limiting value admitted by Regge theory, i.e. one half (fit D):

$$
\alpha_f = 0.5, \qquad \gamma_\Sigma = 0, \qquad \delta_g = 1, \qquad \delta_{NS} = 1, \qquad \text{(fit D)}. \tag{14}
$$

The results of this fit are also shown in table 1. The χ^2 is now much worse, but the physical results do not change much, especially in the nonsinglet sector (for example the central value of g_A is about the same in fits C and D). One could presumably optimize the choice of the initial scale Q_0^2 to make the agreement with Regge theory even better. Note that for all fits A-D the χ^2 per degree of freedom is below 1, but the fit B is neatly preferred [46], while the fit D is much disfavoured in terms of the absolute χ^2 value.

In figs. 2a-c we display the best-fit g_1 (fit B) for protons, neutrons and deuterons at the Q^2 of the data.


dashed: $Q^2=Q^2(x)$ (SMC)
solid: $Q^2=Q^2(x)$ (E143)
dotdashed: $Q^2=1$ GeV^2
dotted: $Q^2=10$ GeV^2
×: SMC
◇: E143
(a)
xg_1^p
x
dashed: $Q^2=Q^2(x)$ (SMC)
solid: $Q^2=Q^2(x)$ (E143)
dotdashed: $Q^2=1$ GeV^2
dotted: $Q^2=10$ GeV^2
×: SMC
◇: E143
(b)
xg_1^d
x

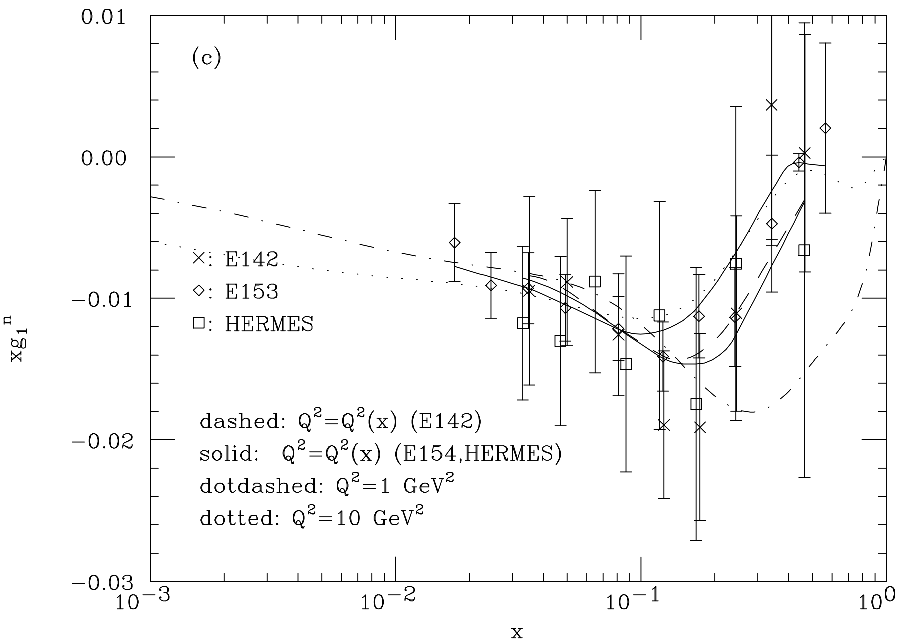
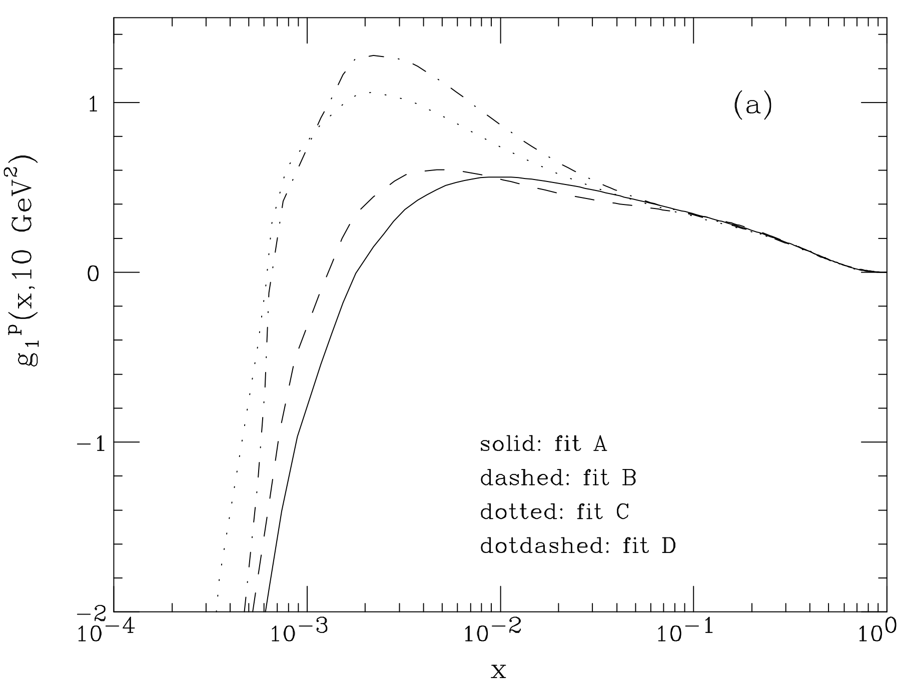
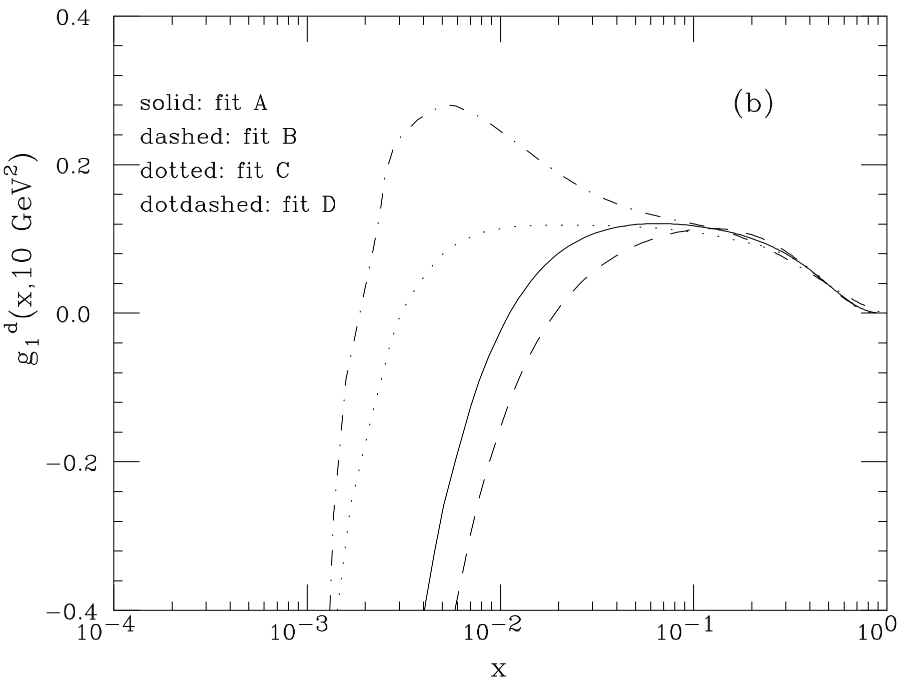


Figure 2: *Plots of $xg_1(x,Q^2)$ for fit B for (a) proton, (b) deuterium, and (c) neutron targets. The data points with total errors are also shown.*





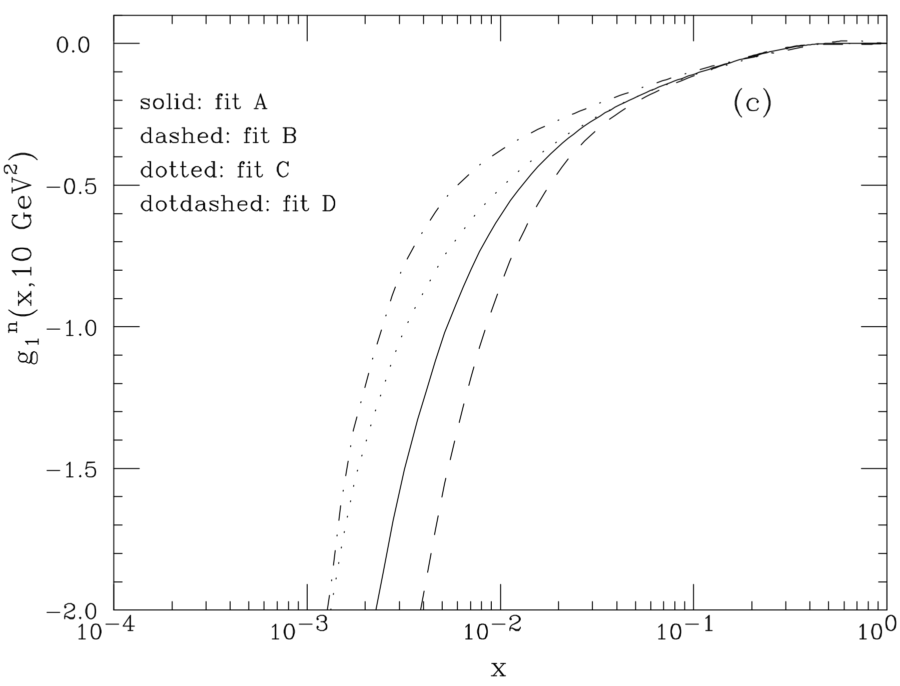


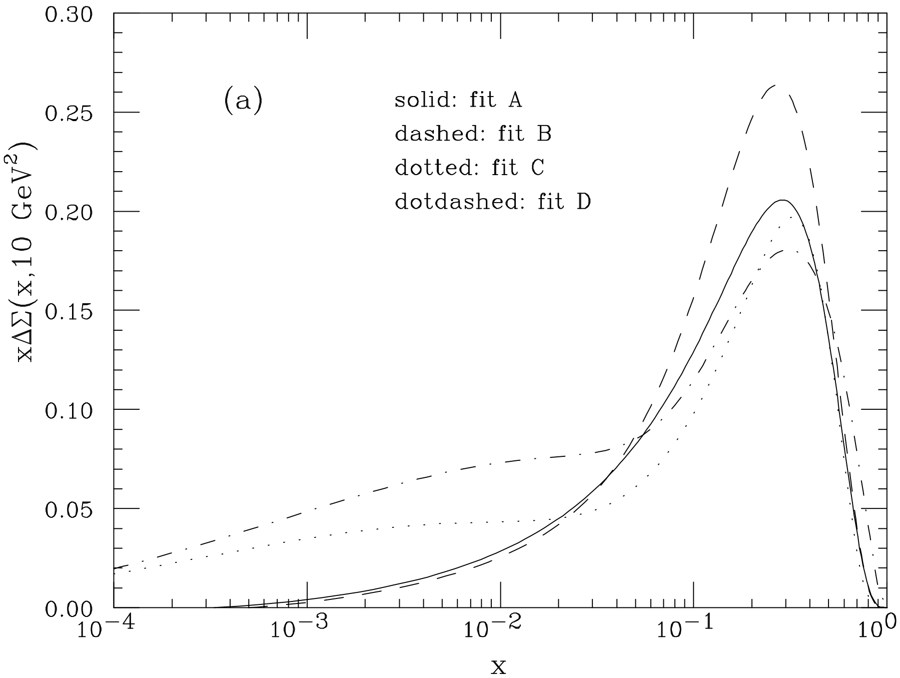
Figure 3: *Plots of $g_1(x, Q^2)$ for (a) proton (b) deuterium (c) and neutron targets for fits A–D at $Q^2 = 10\ GeV^2$.*

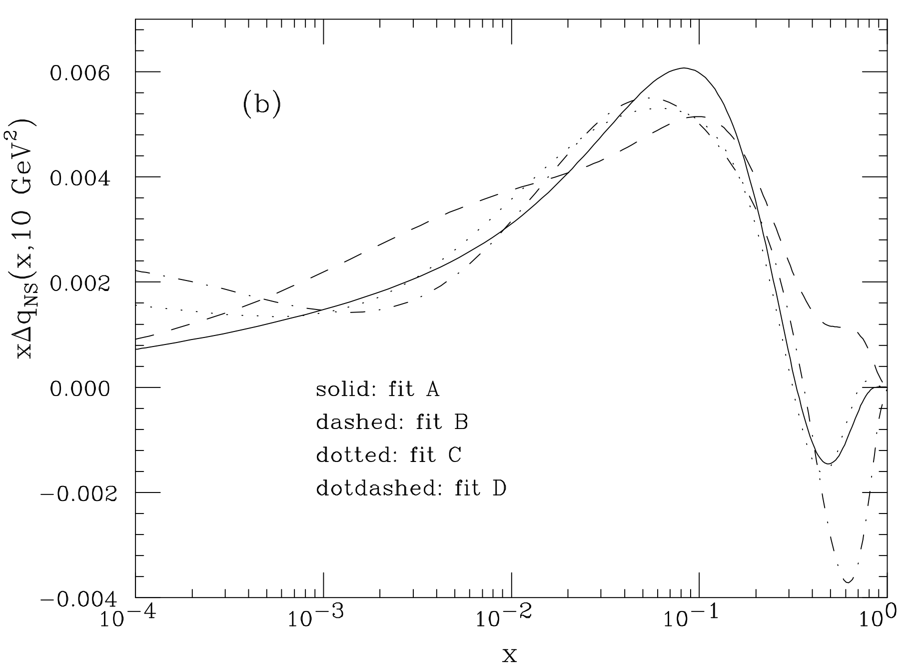
In figs. 3a-c we then compare the best-fit forms of g_1 corresponding to fits A–D at $Q^2 = 10$ GeV^2. The figures show that while the four fits are reasonably close together in the measured region ($x \sim 0.003 - 0.03$ up to $x \sim 0.8$), they become very different in the small x region. Note that at this Q^2 value g_1 has indeed become negative at small x in all cases. In fig. 4a-c we display the resulting polarized parton densities obtained from the fits at the same value of Q^2. Here (as in ref. [21]) Δq_{NS} is the quantity defined in eq. (4) for a deuterium target, rescaled by a factor $< e^2 >= 5/18$ (above charm threshold). Note that the opening of the charm threshold makes the shape of the nonsinglet different for protons and deuterons, as can be seen comparing $x\Delta q_{NS}$ (fig. 4b) with $x\Delta q_3 = x(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d})$, which is displayed in fig. 5. Especially in the singlet sector, the behaviour at small x is quite different in each case: fits C and D develop a particularly robust tail at small x. It is the large positively polarized gluon that drives g_1 negative at small x.

In table 2 we report the values obtained by computing the first moments of g_1 by integration of the four fits, both in the measured range of x and in the whole range at the 'average' values of Q^2 quoted by each experiment on protons, deuterons and neutrons. We see that while the truncated moments are remarkably close to each other the complete moments show a much wider spread. We also report the values of the truncated moments obtained by evolving the data to a common scale by means of the traditional (but unjustified) assumption that the asymmetries are scale independent and then summing over the bins, and those given by the experimental collaborations with their associated total errors. The latter two values should in principle coincide, and only differ because of details in the way g_1 is determined from the measured asymmetries (such as the use of different parameterizations of the unpolarized structure function F_2, or the inclusion of some higher twist corrections, as done by some experimental collaborations). The effect of the Q^2 dependence in the measured region is sizeable but smaller than the experimental error. Much larger is the indirect effect of scaling violations on the extrapolation at small x because of the larger scale dependence at small x.

4 Phenomenological Implications

We will now discuss the quantitative consequences that can be derived from the results of the previous section. In particular we will discuss the status of the Bjorken sum rule, the polarization of the quark flavor singlet combination and of the gluons in the proton, the determination of the singlet axial charge, and the determination of α_s. A detailed estimate of the uncertainties has been given in ref. [21]. We will not repeat the error analysis because the quality of the data is essentially unchanged, and thus use the values of the theory error quoted in ref. [21], to which we refer the reader for details. These values are summarized in table 3. We wish to point out that we have been particularly careful in estimating the theoretical uncertainty related to the choice of functional form adopted for the fits, as well as from the truncation of the perturbative expansion (renormalization and factorization scales variation). As shown in table 3, we find that these are the most important sources of theoretical error.


(a)
solid: fit A
dashed: fit B
dotted: fit C
dotdashed: fit D
x∆Σ(x,10 GeV²)
x


(b)
solid: fit A
dashed: fit B
dotted: fit C
dotdashed: fit D
x∆q_NS(x,10 GeV²)
x

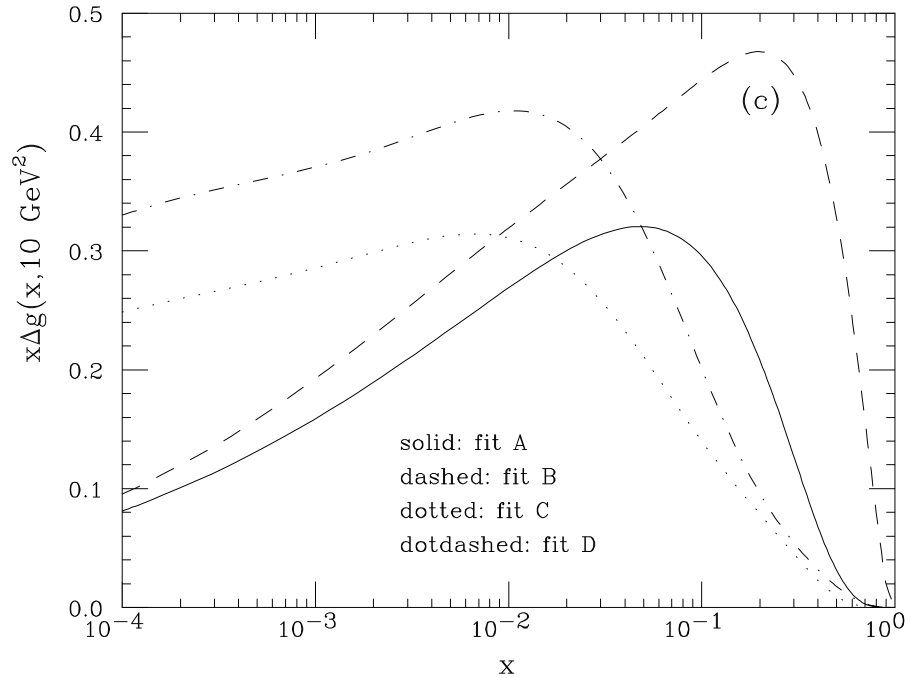


Figure 4: *Polarized quark singlet (a) nonsinglet (b) and gluon (c) distributions for fits A–D at $Q^2 = 10\ GeV^2$.*

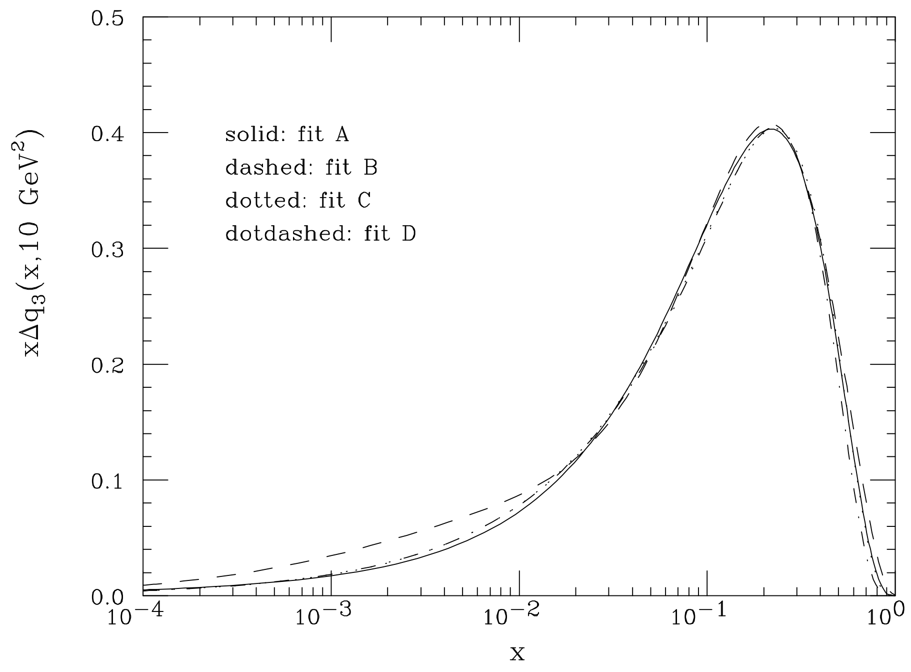


Figure 5: *The nonsinglet density $x\Delta q_3 = x(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d})$ in the proton at $Q^2 = 10GeV^2$ for fits A-D. The area under the curves is directly the axial charge g_A.*

4.1 Testing the Bjorken Sum Rule

One general way to test the Bjorken sum rule is to determine g_A and the associated error from fitting the whole set of available data points. Using the fits described in the previous section and shown in table 1, and the estimate of the uncertainties of ref. [21], we may take as a result

$$g_A = 1.18 \pm 0.05(\text{exp}) \pm 0.07(\text{th}) = 1.18 \pm 0.09, \tag{15}$$

where the central value is obtained as the average between the maximum and minimum values of table 1. This is the procedure we followed in our previous paper, ref. [21]. Alternatively, we can take into account that, contrary to what was found in ref. [21], fit B now has a significantly better χ^2 than any other fit. We can then take the central value from fit B and introduce an asymmetric theoretical error to take into account the lower values of g_A from fits A, C and D. We then obtain

$$g_A = 1.23 \pm 0.06(\text{exp})\,^{+0.06}_{-0.11}(\text{th}) = 1.23\,^{+0.08}_{-0.12}. \tag{16}$$

The fitted value is to be compared with the direct measurement $g_A = 1.257 \pm 0.003$ [47] from β-decay. Thus we find that the Bjorken sum rule is confirmed to within one standard deviation but still with an accuracy of only about 8%.

4.2 Singlet First Moments

Similarly in the singlet sector one obtains from the data values for $\eta_q = \Delta\Sigma(1)$ (the conserved polarized singlet quark density), for $\eta_g = \Delta g(1, 1\,\text{GeV}^2)$ (the first moment of the polarized gluon density evaluated at $Q^2 = 1$ GeV^2), and for $a_0(10\,\text{GeV}^2)$ (the non conserved singlet axial charge defined implicitly from the singlet part of g_1 by eq. (6)). This latter quantity approaches a finite limit at infinite Q^2 because the corresponding anomalous dimension starts at two loops, and within the present accuracy $a_0(10\,\text{GeV}^2)$ is equivalent to $a_0(\infty)$. The values for these three quantities as obtained from our representative fits are reported in table 1. We then studied in detail, following ref. [26], the theoretical errors from the various different sources: results for these are listed in table 3. We find (with the same criteria as for g_A in eq. (15))

$$\begin{aligned}
\Delta\Sigma(1) &= 0.46 \pm 0.04\ (\text{exp}) \pm 0.08\ (\text{th}) = 0.46 \pm 0.09, \\
\Delta g(1, 1\,\text{GeV}^2) &= 1.6 \pm 0.4\ (\text{exp}) \pm 0.8\ (\text{th}) = 1.6 \pm 0.9, \\
a_0(\infty) &= 0.10 \pm 0.05\ (\text{exp})\,^{+0.17}_{-0.10}\ (\text{th}) = 0.10\,^{+0.17}_{-0.11}.
\end{aligned} \tag{17}$$

Alternatively, if we proceed as for g_A in eq. (16), we find

$$\begin{aligned}
\Delta\Sigma(1) &= 0.44 \pm 0.04\ (\text{exp}) \pm 0.08\ (\text{th}) = 0.44 \pm 0.09, \\
\Delta g(1, 1\,\text{GeV}^2) &= 1.4 \pm 0.3\ (\text{exp}) \pm 0.8\ (\text{th}) = 1.4 \pm 0.9, \\
a_0(\infty) &= 0.11 \pm 0.05\ (\text{exp})\,^{+0.23}_{-0.07}\ (\text{th}) = 0.11\,^{+0.23}_{-0.09}.
\end{aligned} \tag{18}$$

The close compatibility of the results (17) and (18) is a reflection of the remarkable stability of the first moments in the four independent fits A-D. The parameter $a_0(\infty)$ measures the degree of 'spin crisis': the singlet axial charge of the nucleon is still compatible with zero as it was at the beginning of the story [1]. Note that with the naive Regge extrapolation at small x the experimental result for the axial singlet charge would be significantly larger, with a much smaller error: for example in ref. [48] a value $a_0(\infty) = 0.33 \pm 0.04$ was quoted. There is also some evidence for a positive gluon polarization in the nucleon (increasing with Q^2 as $1/\alpha_s(Q^2)$). The amount of gluon polarization is large enough to allow the first moment $\Delta\Sigma(1)$ of the conserved singlet quark density to get close to $a_8 \sim 0.58$, which in the absence of all SU(3) and chiral symmetry breaking effects, could be identified with the constituent spin fraction [49]. This can be seen as a direct confirmation of the physical explanation of the 'spin crisis', advocated in refs. [10, 12, 13], as due to the anomaly and well described in terms of the QCD parton model (for more general possibilities, see refs. [14]).

Whereas the data allow a good determination of the singlet and triplet quark components, we have verified explicitly that with present data it is still not possible to separate off the octet quark component and thus determine the value of a_8. In fact the quality of the fit is essentially unchanged if a_8 is varied by 30% around its central value. The effects of separating the octet components will be further discussed in sect. 5.

4.3 Determination of α_s

The above discussion on g_A makes it clear that the determination of α_s from the Bjorken integral is adversely affected by the increased ambiguity from the small x extrapolation that follows from the demise of the naive Regge behaviour at small x. Fortunately we find that α_s can be determined directly from the available data without extrapolation in the small x region if the totality of the data is taken into account and not just the Bjorken integral. The value of α_s is then determined by the strong scaling violations needed to accommodate the difference between the data at small Q^2 from the SLAC experiments and those at larger Q^2 from the SMC in the common range of x. While $\Delta g(1, 1\,\mathrm{GeV}^2)$ is mainly fixed by the proton data, α_s is determined by the difference between proton and neutron.

To show this we repeated the fits A-D but fixing g_A to its experimental value and instead fitting α_s. In all cases the central value was found to be close to $\alpha_s(m_Z) = 0.120$. Since the various fits differ considerably in the unmeasured region, this shows that it is the behaviour in the measured region that matters. In addition, the Bjorken integral is appreciably different in the different fits A-D (and the value of $\Delta g(1, 1\,\mathrm{GeV}^2)$ even more so) but this difference does not affect α_s very much. Furthermore, the resolution of the discrepancy between the fitted value of g_A eq. (15) and its experimental value would require a much larger increase of α_s if only the Bjorken integral were relevant for fixing α_s. These results show that α_s is much better constrained by the overall pattern of scaling violations than by the Bjorken integral alone. The theoretical uncertainties that affect this determination of α_s are listed in table 3. The main source of uncertainty originates from higher order and higher twist contributions.

We thus obtain finally

$$\alpha_s(m_Z) = 0.120\,^{+0.004}_{-0.005}(\mathrm{exp})\,^{+0.009}_{-0.006}(\mathrm{th}) = 0.120\,^{+0.010}_{-0.008}. \qquad (19)$$

This reasonably good determination of $\alpha_s(m_Z)$ could still be improved with better data: it is important to notice that without the very recent neutron data [8] the experimental error would be twice as large. We see no reason why it could not be as good as the determination from unpolarized data if more data were added and the experimental errors consequently reduced. However the theoretical error is already the dominant one; it could also be reduced by more data because it is made particularly large by the small Q^2 values of the neutron measurements.

5 Comparison with Related Work

Recently, after our previous work in ref. [21], a number of NLO analyses of the available data on polarized deep inelastic scattering have been published [50, 51, 52, 53]. The results of ref. [53] are essentially consistent with ours. Here we briefly comment on the first two papers and their conclusion. These two papers have many starting points in common and their conclusions also look similar.

The E154 Collaboration has presented in ref. [50] its own complete NLO analysis of all available data. What is particularly interesting is that their final answers to the main questions seem to be somewhat different from ours, even though there should be no real contradiction provided all the various uncertainties are invoked in a conservative form.

Apart from the determination of $\alpha_s(m_Z^2)$, an issue they do not address, one first main question is the quantitative assessment of the validity of the Bjorken sum rule. Then there is the determination of the gluon density with special emphasis on the magnitude of the first moment and the question of whether the gluon first moment can explain the apparent difference between the singlet quark moment defined from g_1 and the naive constituent quark expectation.

In fact their result for g_A is somewhat low: for example, in the AB scheme (which we also use), they find $g_A = 1.07^{+0.12}_{-0.06}$. Similarly the value of the gluon first moment at $Q^2 = 5$ GeV^2 in the AB scheme is found to be $\Delta g(1, 5\,\mathrm{GeV}^2) = 0.4^{+1.7}_{-0.9}$, while in the $\overline{\mathrm{MS}}$ scheme[2] they obtain $\Delta g(1, 5\,\mathrm{GeV}^2) = 1.8^{+0.7}_{-1.0}$ (note that Δg should be the same in the two schemes at NLO accuracy). Translated to $Q^2 = 1$ GeV^2 their gluon result in the $\overline{\mathrm{MS}}$ scheme becomes $\Delta g(1, 1\,\mathrm{GeV}^2) = 1.12^{+0.5}_{-0.85}$ compared with our values given in eq. (17). Thus their results on the gluon size are quite inconclusive, while we find a moderate indication for a large positive gluon.

We have emphasized in our work the importance of using a sufficiently general input parameterization for the determination of the final results. The input form used in ref. [50] is different from any of ours: there, the polarized densities are assumed to be proportional to the unpolarized ones with a proportionality factor given by powers of x and $(1-x)$. The initial

[2] Notice that in this context the definition of the $\overline{\mathrm{MS}}$ scheme is not unique: in fact, a consistent definition of the γ_5 matrix in dimensional regularization requires the introduction of finite counterterms in order to restore the conservation of nonsinglet axial currents. Here, as is customary in the literature, by $\overline{\mathrm{MS}}$ we mean the factorization scheme adopted in ref. [16]. An important difference between this scheme and the AB scheme is that in the AB scheme $\Delta\Sigma(1)$ is scale independent, whereas in the scheme of ref. [16] it coincides with $a_0(Q^2)$. The relation between the singlet quark first moments in the two schemes is thus given by eq. (2).

value of Q_0^2 is very low: $Q_0^2 = 0.34$ GeV^2. Moreover, the two nonsinglet combinations $\Delta u - \Delta d$ and $\Delta u + \Delta d - 2\Delta s$ are parametrized independently of each other, while in our framework they are assumed to have the same shape in x. In order to check whether this can be the origin of the differences between our results and those of ref. [50], we have repeated our fits with independent parametrizations for $\Delta u - \Delta d$ and $\Delta u + \Delta d - 2\Delta s$, keeping however the first moment of $\Delta u + \Delta d - 2\Delta s$ fixed to its measured value, 0.579. This does not seem to produce any sizeable effect: for example, fit B gives

$$
\begin{aligned}
\Delta\Sigma(1) &= 0.45 \;\; (0.44 \pm 0.05), && (20)\\
\Delta g(1, 1\,\mathrm{GeV}^2) &= 1.5 \;\; (1.4 \pm 0.5) && (21)\\
a_0(\infty) &= 0.10 \;\; (0.11 \pm 0.07) && (22)\\
g_A &= 1.23 \;\; (1.23 \pm 0.05), && (23)
\end{aligned}
$$

where we have indicated in brackets the values obtained in the previous section (with $\Delta u - \Delta d$ and $\Delta u + \Delta d - 2\Delta s$ proportional to each other) and the error corresponding to our estimate of the total error due to the parametrization (from table 3). The quality of the fit is unchanged. We therefore conclude that introducing a separate parametrization of the octet $\Delta u + \Delta d - 2\Delta s$ does not affect our results. In other words, by taking the octet and triplet to be proportional we had introduced no bias, and thus the errors on physical quantities coming from differences in the input parametrizations were correctly estimated in ref. [21].

Another important point is the choice of the factorization scheme. Chosing a different scheme induces an uncertainty in the fitted parameters which is formally of next-to-next-to-leading order. We estimated the error originated by the truncation of the perturbative series by changing the value of the renormalization and factorization scales (which corresponds to a modification of the finite parts of the counterterms), and we included this large uncertainty in our estimate of the total error on each quantity.[3] In order to check that this estimate is correct, we repeated our fits in the $\overline{\mathrm{MS}}$ scheme, and we found that indeed the central values of the relevant quantities are within the estimated uncertainties. For example, fit B in the $\overline{\mathrm{MS}}$ scheme gives

$$
\begin{aligned}
\Delta g(1, 1\,\mathrm{GeV}^2) &= 1.2 \;\; (1.4 \pm 0.6) && (24)\\
a_0(\infty) &= 0.15 \;\; (0.11\,^{+0.15}_{-0.07}) && (25)\\
g_A &= 1.23 \;\; (1.23 \pm 0.03), && (26)
\end{aligned}
$$

where again we have indicated in brackets the values of our original fit B, and the estimated error due to scheme change (from table 3). Notice that, since in the $\overline{\mathrm{MS}}$ scheme $\Delta\Sigma(1, Q^2) = a_0(Q^2)$, eq. (25) shows that the first moment of the quark distribution found in the two schemes agree to NLO accuracy provided they are transformed using eq. (2). Also in this case, we do not see any significant deviation due to the difference in the scheme choice, so again we conclude that, while our estimate ot the scheme-dependence uncertainty was correct, this is not the source of the discrepancy with ref. [50].

[3] Notice that factorization scale variations were not considered in ref. [50], so that the contribution to the theoretical error due to scheme dependence was not included there.

The two analyses, however, do not only differ in the choice of input ansatz. We observe a number of features in the analysis of ref. [50] that we find unconvincing. First they apply the positivity relations $|\Delta f| \leq f$ down to their very low initial value of $Q_0^2 = 0.34$ GeV^2. We have already remarked that the positivity relations are not generally true at NLO and the error induced by using them might in principle be very large at small Q^2. What are bounded by positivity constraints are a number of physical quantities like asymmetries or cross sections. Only at leading order do the bounds on these physical quantities translate into the simple inequality $|\Delta f|/f \leq 1$. At small Q^2 there are large corrections that depend on the precise definition of the parton densities. The authors of ref. [50] find that the positivity constraint is very powerful. In our opinion this means that they have introduced a large bias. Other possible biases are introduced by the fact that they separate valence and sea, which cannot, even in principle, be disentangled by measurements of g_1; this separation is, so to say, inherited from the assumed proportionality to unpolarized densities. Furthermore their parametrisation implies that, in the interesting region of parameters, the singlet quarks necessarily dominate over the nonsinglet, while we find the opposite. Although they get a reasonably good fit in the measured region, still the fact that we show that a different region of lower minimum χ^2 exists which they cannot access shows again that a bias has been introduced. Also, they use a value of α_s which is very low and by now completely obsolete: $\alpha_s(m_Z) = 0.109^{+0.007}_{-0.001}$. We have checked explicitly that such a small value of α_s induces an overestimate of the singlet axial charge a_0, and an underestimate of g_A. For all these reasons we are not convinced that the E154 fit is really so significant or representative.

The work in ref. [51] is a summary and update along the lines of previous analyses [54]. This analysis has some features in common with the E154 approach we have just described. In fact the same parametrization of the polarized densities as proportional to the unpolarized ones is adopted, and the initial scale is the same: $Q_0^2 = 0.34$ GeV^2, thus again, in particular, enforcing a positivity constraint down to this very low initial scale. Therefore, their fit suffers from the same unjustifiable bias as that of E154. Furthermore, the fits of ref. [51] have been performed including also data points with 0.6 GeV$^2 \leq Q^2 \leq$1 GeV^2. This is undeniably questionable in the context of a perturbative analysis, especially because data at small Q^2 are usually taken in the small-x region, where the effect of evolution is very important.

The issues of testing the Bjorken sum rule and of measuring α_s were not addressed in ref. [51]. The Bjorken sum rule is imposed, and a fixed value of $\alpha_s(m_Z) = 0.109$ is input (again very low). The analysis is performed in the $\overline{\mathrm{MS}}$ scheme. What is interesting is that several input forms for the gluon are studied and compared. For example, one form starts with $\Delta g = g$ at the input scale, another with $\Delta g = 0$ and they are compared with the best fit form of Δg. With the first option one ends up with a rather large polarized gluon density while the last one leads to a negligible amount of polarized gluons. The best fit is intermediate, with a moderate value of the first moment: $\Delta g(1, 10\,\mathrm{GeV}^2) \sim 1.45$. The three χ^2 values are not much different: $\chi^2 = 127.44, 123.02, 124.24$ for the large gluon, the best fit and the small gluon, respectively. Thus the small gluon is only about 1 σ away from the best fit. The conclusion of the paper is that the data do not allow to derive any consequence on the size of the gluon component, and in particular that a small gluon is perfectly compatible with the data.

We observe that their best fit value for the first moment is somewhat on the low side. However, we believe that several sources of bias have been introduced in this analysis. We can test this by comparing the best-fit χ^2 of the present analysis to ours. To this purpose, notice that the data of refs. [1] and [22], as well as the low Q^2 data which are included in the fits of ref. [51] (40 data points, overall), have very large errors and thus lower considerably the χ^2 per degree of freedom. We do not consider the inclusion of low Q^2 points to be acceptable, but if we were to include the data of ref. [1, 22] with $Q^2 \geq 1$ Gev^2 (22 data points) our best fit (fit B) would drop to χ^2/d.o.f.=0.74, to be compared to the value χ^2/d.o.f.=0.78 of ref. [51]. Therefore, it is true that, given the size of the existing errors, a nearly vanishing first moment of Δg is within 1 σ from the central value found in ref. [51]; however, such a small central value appears to be considerably biased, and indeed it appears to correspond to a significantly larger [46] value of the total χ^2. Therefore, we can say that if one samples over a larger set of input parametrisations the central value is actually larger; the result of [51] is compatible with ours, but artificially displaced on the low side.

In summary, while we agree that the determination of the polarized gluon density from scaling violations is affected by large uncertainties, we claim that we have examined a wide range of starting parametrizations in our analysis and thereby avoided some of the more questionable assumptions and biases that are often found in the literature. On the basis of our more systematic work we have presented some moderate evidence for a large and positive gluon component. Furthermore, we have shown that the value of the singlet axial charge is affected by a substantially larger error (while the central value is smaller) than usually claimed.

6 Conclusion and Outlook

In the present paper we have presented an update of our recent analysis of all existing data on the polarized structure function g_1 and a comparison with other similar analyses. We have addressed the main questions of the validity of the Bjorken sum rule, of the determination of the strong coupling and of the measurement of the polarized gluon density. Overall we find a remarkable consistency of data and theory: the Bjorken sum rule is valid within the existing errors, the value of α_s extracted from the data is in good agreement with the world average for this quantity and we find some evidence for a large and positive gluon component that could perhaps be large enough to explain the difference between parton and constituent quarks. It is true however that the determination of the gluon density from the observed scaling violations is the most ambiguous and controversial aspect of the analysis. A substantial improvement of our knowledge of the polarized gluon density could be obtained through better and more extended measurements of g_1 at small x as could be made at HERA with polarized proton beams [55]. In addition, more direct information on the polarized gluon can be obtained from the study of additional hard processes beyond totally inclusive deep inelastic scattering, as is planned at COMPASS [56], at HERA with polarized proton beams and at RHIC. The preparation of this new phase of the study of polarized structure functions is now actively in progress. Several different strategies have been discussed for the determination of the gluon polarization from experiment [57]-[63]. Most of the early analyses were limited to leading order, but the corresponding work on extending these analyses at the next to leading order is now in progress [64]-[67]. The prospects for the future are exciting and the field will remain of great interest in the years to come.

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Parameters	A	B	C	D
d.o.f.	$123-10$	$123-11$	$123-8$	$123-8$
Q_0^2/GeV^2	1	1	0.3	0.3
η_Σ	0.405 ± 0.032	0.440 ± 0.037	0.420 ± 0.022	0.516 ± 0.031
α_Σ	0.507 ± 0.169	1.726 ± 0.433	3.156 ± 1.028	0.5 (fixed)
β_Σ	3.232 ± 0.575	2.748 ± 0.472	3.764 ± 1.332	0.816 ± 0.194
γ_Σ	3.006 ± 1.862	0 (fixed)	0 (fixed)	0 (fixed)
η_g	0.949 ± 0.185	1.415 ± 0.322	0.506 ± 0.094	0.687 ± 0.099
α_g	-0.486 ± 0.281	3.174 ± 1.494	0.155 ± 0.289	0.5 (fixed)
β_g	4 (fixed)	1.032 ± 0.535	15 (fixed)	11.58 ± 5.69
γ_g	3.006 ± 1.862	47.0 ± 103.1	0 (fixed)	-0.541 ± 4.896
g_A	1.140 ± 0.043	1.232 ± 0.057	1.142 ± 0.030	1.121 ± 0.029
a_8	0.579 (fixed)	0.579 (fixed)	0.579 (fixed)	0.579 (fixed)
α_{NS}	-0.576 ± 0.049	1.662 ± 0.144	0.874 ± 0.220	0.5 (fixed)
β_{NS}	2.668 ± 0.218	5.399 ± 0.208	2.325 ± 0.421	3.115 ± 0.331
γ_{NS}	34.36 ± 17.64	-0.214 ± 0.088	0 (fixed)	9.869 ± 8.739
χ^2	96.4	90.5	97.6	110.5
χ^2/d.o.f.	0.853	0.808	0.849	0.960
$\Delta g(1, 1\text{GeV}^2)$	0.95 ± 0.18	1.41 ± 0.32	1.67 ± 0.31	2.20 ± 0.32
$a_0(10\text{GeV}^2)$	0.18 ± 0.04	0.11 ± 0.05	0.04 ± 0.04	0.02 ± 0.03

Table 1: Results of fits A–D described in the text

	SMC : p	E143 : p	SMC : d	E143 : d	E142 : n	E154 : n
$\langle Q^2 \rangle/\text{GeV}^2$	10	3	10	3	2	5
Meas. Range: Exp.	0.139	0.1200	0.0407	0.0400	-0.0280	-0.0360
Exp. Error	± 0.01	± 0.0089	± 0.0069	± 0.0050	± 0.0085	± 0.0064
A_1 ind. Q^2	0.1478	0.1041	0.0443	0.0394	-0.0290	-0.0362
Meas. Range: A	0.1257	0.1080	0.0444	0.0393	-0.0317	-0.0319
Meas. Range: B	0.1247	0.1039	0.0404	0.0346	-0.0361	-0.0364
Meas. Range: C	0.1276	0.1085	0.0458	0.0402	-0.0293	-0.0318
Meas. Range: D	0.1309	0.1079	0.0505	0.0410	-0.0304	-0.0287
Full Range: A	0.1175	0.1142	0.0301	0.0292	-0.0551	-0.0566
Full Range: B	0.1169	0.1130	0.0224	0.0210	-0.0703	-0.0716
Full Range: C	0.1021	0.0981	0.0145	0.0128	-0.0720	-0.0728
Full Range: D	0.0970	0.0926	0.0110	0.0089	-0.0745	-0.0749

Table 2: Determination of the first moment $\Gamma_1(\langle Q^2 \rangle)$ eq. (6). For each experiment we display the average value of Q^2 and the contribution to the first moments from the measured range of x, as given, first, by the experimental collaborations, with the corresponding total (statistical and systematic) error, then by summing over experimental bins while evolving the data assuming scale independent asymmetries, and finally as obtained from integration of the fits A–D. In the last four rows the complete first moments obtained from the fits A–D are shown.

	g_A	$\Delta\Sigma$	Δg	a_0	α_s
experimental	± 0.05	± 0.04	± 0.4	± 0.05	${}^{+0.004}_{-0.005}$
fitting	± 0.05	± 0.05	± 0.5	± 0.07	± 0.001
α_s & a_8	± 0.03	± 0.01	± 0.2	± 0.02	± 0.000
thresholds	± 0.02	± 0.05	± 0.1	± 0.01	± 0.003
higher orders	± 0.03	± 0.04	± 0.6	${}^{+0.15}_{-0.07}$	${}^{+0.007}_{-0.004}$
higher twists	± 0.03	-	-	-	± 0.004
theoretical	± 0.07	± 0.08	± 0.8	${}^{+0.17}_{-0.010}$	${}^{+0.009}_{-0.006}$

Table 3: Contributions to the errors in the determination of the quantities g_A, $\Delta\Sigma(1)$, $\Delta g(1, 1\mathrm{GeV}^2)$, $a_0(\infty)$ and $\alpha_s(m_Z)$ from the fits described in the text.